Statistical Methods for Plant Biology

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Anirudh V. S. Ruhil

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The Voinovich School of Leadership and Public Affairs

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Making and Using Hypotheses

Hypothesis Testing

Definition

Hypothesis testing is an inferential procedure that uses sample data to evaluate the credibility of a hypothesis about a population parameter. The process involves ...

- A hypothesis – an assumption that can neither be proven nor disproven
- Hypotheses are denoted by H, for example ...
  1. H: At most 5% of GM trucks breakdown in under 10,000 miles
  2. H: Heights of adult males is distributed with $\mu = 72$ inches
  3. H: Mean annual temperature in Athens (OH) is > 62
  4. H: 10% of Ohio teachers are "Accomplished"
  5. H: Mean county unemployment rate is 12.1%
  6. H: Mean undulation rate of Gliding Snakes is 1.375 Hz
  7. H: Radiologists are as likely to have sons as daughters

The Null and The Alternative Hypotheses

- Null Hypothesis (H_0) is the statement believed to be true
- Alternative Hypothesis (H_A) is the statement believed to be true if (H_0) is rejected
  1. H_0: The density of dolphins *does not differ* across areas with/without drift-net fishing
     H_A: The density of dolphins *does differ* across areas with/without drift-net fishing
  2. H_0: The antidepressant effects of Sertraline *do not differ* from those of Amitriptilyne
     H_A: The antidepressant effects of Sertraline *do differ* from those of Amitriptilyne
  3. H_0: Brown-eyed parents, each of whom had one parent with blue eyes, *have* brown- and blue-eyed children in a $3:1$ ratio
     H_A: Brown-eyed parents, each of whom had one parent with blue eyes, *do not have* brown- and blue-eyed children in a $3:1$ ratio
- Note that they are (a) Mutually Exclusive: Either H_0 or H_A is True; and (b) Exhaustive: H_0 and H_A exhaust the Sample Space

Two-Tailed and One-Tailed Hypotheses

- Two-Tailed hypotheses assume the following structure:
  H_0: Mean body temperature of humans is $= 98.6^0$F
  H_A: Mean body temperature of humans is $\neq 98.6^0$F
- One-Tailed hypotheses assume the following structure:
  H_0: Mean body temperature of humans is $\leq 97^0$F
  H_A: Mean body temperature of humans is $> 97^0$F
  or
  H_0: Mean body temperature of humans is $\geq 99^0$F
  H_A: Mean body temperature of humans is $< 99^0$F

Rule-of-thumb: Setup the Null hypothesis as the one to be rejected

Hypothesis Testing Protocol

1. State H_0 and H_A
2. Collect your sample data
3. Calculate the estimate of interest (for e.g., the sample mean)
4. Draw your conclusion ... should we Reject H_0 or Fail to Reject H_0? But how?
    - Under H_0 being TRUE, we would expect $\bar{Y} = \mu$
    - If $\bar{Y} \neq \mu$ then we must realize this could happen (1) due to chance or (2) it could well be that H_0 is not true
    - The farther apart is $\bar{Y}$ from μ, the more likely it is that H_0 is NOT TRUE
    - We thus set a bar: "We can say H_0 is not true if the probability of getting $\bar{Y}$ as far or farther from μ, by chance alone, is less than some probability that we get to pick (α)"
    - Conventionally, the bar is set at 0.05 or 0.01 ... i.e., we Reject H_0 if, assuming H_0 to be true, the probability of observing our sample-based estimate, by chance alone, is $\leq \alpha$

The Decision Calculus Revisited

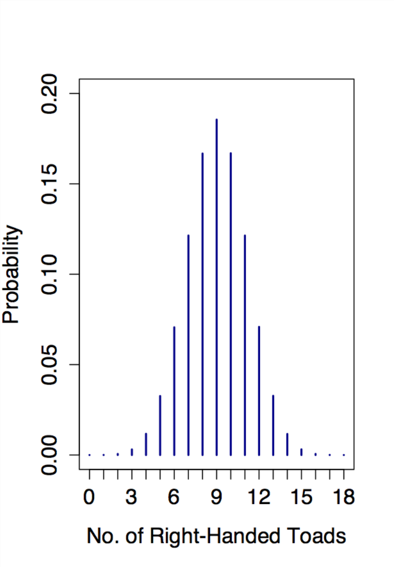
1. Clearly state H_0 and H_A
2. Choose α (called the Significance Level)
3. Draw your sample
4. Calculate the estimate of interest (i.e., Mean, Standard Deviation, Proportion, etc.)
5. See how likely it would be to obtain this estimate of interest *if H_0 were true* ... i.e., calculate the probability (called the P-value) of observing this estimate of interest
6. Reject H_0 if the P-value $\leq \alpha$
   Do Not Reject H_0 if the P-value $> \alpha$

Hypothesis Testing: An Example

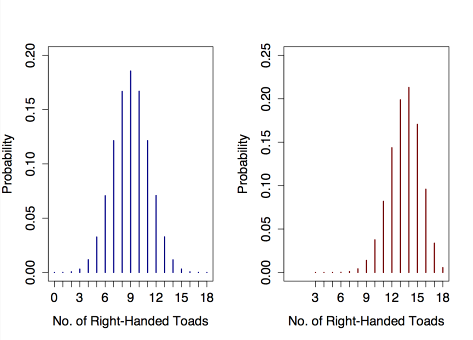
Toads and Handedness: An Example

1. Are toads right-handed or left-handed? We don't know so our best guess might be that H_0 : left-handed toads occur as often as right-handed toads $(p = 0.5)$. If this statement is untrue then it must be that H_A : left-handed toads do not occur as often as right-handed toads $(p \neq 0.5)$
2. Note that we have a two-tailed test and H_A says we may have a majority or minority of right-handed toads.
3. We collect a random sample of 18 toads. If H_0 is true then we should see 9 right-handed toads. However we find 14 right-handed toads, i.e., $\hat{p} = \frac{14}{18} = 0.78$
4. ... This could happen by chance or because H_0 is not true
5. Let us plot the Null Distribution: the sampling distribution of outcomes for a test statistic under the assumption that H_0 is true. This distribution will be akin to flipping a coin and counting $Heads$, treating $Heads$ as equal to finding a right-handed toad
6. Let us also decide to *Reject* H_0 if we find the probability of $\hat{p}$ to be very low under the null distribution. Say we set the bar at $\alpha = 0.05$
   That is, we have decided to Reject H_0 if the probability (P-value) of seeing 14 or more right-handed toads out of 18 randomly sampled toads is ≤ 0.05

# Right-Handed	Probability
0	0.000004
1	0.000066
2	0.000590
3	0.003089
4	0.011726
5	0.032644
6	0.070677
7	0.121430
8	0.166782
9	0.185549
10	0.166941
11	0.121420
12	0.070888
13	0.032748
14	0.011639
15	0.003132
16	0.000599
17	0.000071
18	0.000004



- Now, if H_0 were true, what should you have expected to see as $\hat{p}$? ... 9
- If H_0 were true we would have seen $\hat{p} \geq 14$, by chance, with a probability of 0.0155 (i.e., P-value of our sample proportion $\hat{p}$) is 0.0155
- It is a two-tailed test so we double it to get $2 \times 0.0155 = 0.031$
- So you have to decide; are you willing to accept that chance dealt you the sample you have or is it that H_0 is in fact not true?
- Since P-value of 0.031 < 0.05 we Reject H_0 ... but we also recognize that this decision could be a mistake!



Errors in Hypothesis Testing

Type I and Type II Errors

	Population Condition	
Decision	H_0 True	H_0 False
Reject H_0	**Type I Error**	No Error
Do Not Reject H_0	No Error	**Type II Error**

- Probability of committing a Type I error $\alpha =$ Level of Significance
- The Power of a test is the probability of rejecting a false H_0, and tests with greater power are preferred to all other tests. The power of a test is difficult yet possible to calculate but in most cases a simple rule suffices: larger samples yield greater power
- Note the language ... "Reject H_0" versus "Do Not Reject H_0" ... the word "Accept" has a ring of finality to it that is unwarranted by statistics

When the Null Hypothesis is Not Rejected

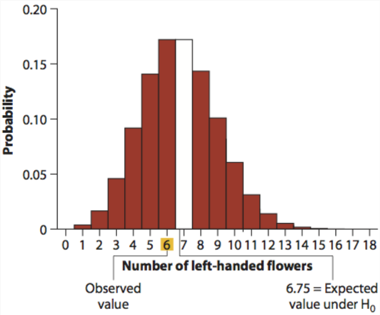
The genetics of mirror-image flowers

Individuals of most plants are hermaphrodites and hence prone to inbreeding. The mud plantain has an avoidance mechanism – the male and female organs deflect to the opposite sides. Bees visiting a left-handed plant are dusted with pollen on their right side which is then deposited only on right-handed plants.

To study the genetics of this variation, a research team crossed pure strains of left- and right-handed flowers to yield only right-handed plants. They then crossed these right-handed plants with each other. A simple model of inheritance would suggest that the offspring of these latter crosses should consist of left- and right-handed flowers in a $1:3$ ratio. In a poll of 27 offspring from one such cross they found six left-handed flowers and 21 right-handed flowers. Do these data support the simple genetic model of inheritance?

When the Null Hypothesis is Not Rejected

- The genetics of mirror-image flowers ... Jesson and Barrett (2002)
- $H_0 : p = \frac{1}{4}$; $H_A : p \neq \frac{1}{4}$
- Let us stay with $\alpha = 0.05$
- Sample yielded $\hat{p} = \frac{6}{27} = 0.2222$
- Simulate distribution under H_0



- We expected to see $\hat{p} = \frac{1}{4} \times 27 = 0.25 \times 27 = 6.75$
- How likely is it that we would end up with $\hat{p} = 0.2222$ if H_0 were true?
- Two-Tailed test so add $P(0) + P(1) + \cdots + P(6) \approx 0.471$
- Since it is a Two-Tailed test, multiply by 2 $= 2 \times 0.471 = 0.942$
- This value *is not* ≤ 0.05 so we Fail to reject H_0
- What does this mean? ... that the sample provides insufficient evidence to reject H_0
- So we do not overturn the Null Hypothesis for now; maybe another study down the road will, and then we will need to articulate a new genetic model

No.of Left-Handed Flowers	Probability
0	0.000418
1	0.003791
2	0.016546
3	0.045907
4	0.091864
5	0.140516
6	0.171938
7	0.171830
8	0.143238
9	0.100775
10	0.060334
11	0.031186
12	0.013902
13	0.005322
14	0.001750
15	0.000518
16	0.000130
17	0.000028
18	0.000005
19	0.000001
$\vdots$	$\vdots$
27	$\vdots$

One-Sided (aka One-Tailed) Tests

One-Tailed (aka One-Sided Tests

1. $H_0 : \mu \leq 0; H_A : \mu > 0$
2. $H_0 : \mu \geq 0; H_A : \mu < 0$
3. $H_0 : p \leq 0; H_A : p > 0$
4. $H_0 : p \geq 0; H_A : p < 0$
5. $H_0 : P \leq 0.50; H_A : p > 0.50$

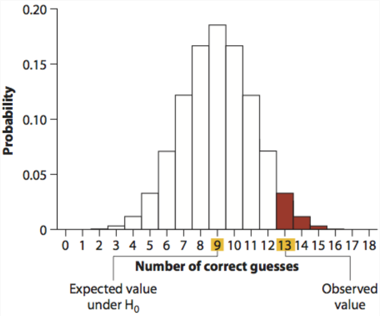
Do daughters resemble their fathers?

In a trial, each participant examines photos of one girl, the father, and another man. They must guess which of the two men is the father. If there is no resemblance, the chance of a correct guess is $50:50$

- Back to the left-handed versus right-handed toads ...
- Sample has $n = 18$
- If H_0 were true $\hat{p} = 0.50$
- In the sample, $\hat{p} = \frac{13}{18} = 0.7222$
- If H_0 were true, how likely would we be to get our 0.7222 by chance alone?

H_0: Participants pick correctly at most 50% of the time $(p \leq 0.50)$
H_A: Participants pick correctly more than 50% of the time $(p > 0.50)$

No. of Correct Guesses	Probability
0	0.000003
1	0.000070
2	0.000582
3	0.003105
4	0.011696
5	0.032646
6	0.070798
7	0.121347
8	0.166896
9	0.185484
10	0.166893
11	0.121574
12	0.070786
13	0.032671
14	0.011674
15	0.003120
16	0.000582
17	0.000069
18	0.000004



What is $p(13$ or more correct guesses$)$? ... 0.048

Since this $P-value$ is < 0.05 we could reject H_0, the Null hypothesis; the data suggest that daughters do seem to resemble their fathers